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THE 67 HZ FEATURE IN THE BLACK HOLE CANDIDATE GRS 1915+105 AS A POSSIBLE "DISKOSEISMIC" MODE

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ABSTRACT

The *Rossi X-ray Timing Explorer* (*RXTE*) has made feasible for the first time the search for high-frequency ($\gtrsim$ 100 Hz) periodic features in black hole candidate (BHC) systems. Such a feature, with a 67 Hz frequency, recently has been discovered in the BHC GRS 1915+105 (Morgan, Remillard, & Greiner). This feature is weak (rms variability $\sim 0.3\% - 1.6\%$), stable in frequency (to within ~ 2 Hz) despite appreciable luminosity fluctuations, and narrow (quality factor $Q \sim 20$). Several of these properties are what one expects for a "diskoseismic" $g-$mode in an accretion disk about a $10.6\,M_\odot$ (nonrotating)$-36.3\,M_\odot$ (maximally rotating) black hole (if we are observing the fundamental mode frequency). We explore this possibility by considering the expected luminosity modulation, as well as possible excitation and growth mechanisms—including turbulent excitation, damping, and "negative" radiation damping. We conclude that a diskoseismic interpretation of the observations is viable.

Subject headings: black hole physics — X-rays: Stars

1. INTRODUCTION

In a series of previous papers (Nowak & Wagoner 1992; Nowak & Wagoner 1993; Perez et al. 1997; hereafter NW92, NW93, P97, respectively), we have discussed a class of modes in thin, Keplerian accretion disks that only exist in the presence of strong-field gravity (not in Newtonian gravity), some of whose eigenfrequencies depend primarily upon the mass and angular momentum of the black hole. The modes are trapped by general-relativistic modification of the radial (κ) and vertical ($\Omega_\perp$) epicyclic frequencies of free particle, circular orbit perturbations (P97, and references therein).

These modes of oscillation are perturbations of the disk which are proportional to $\exp[i(\sigma t + m\phi)]$. With disk angular velocity $\Omega(r)$, their corotating frequency is $\omega(r) = \sigma + m\Omega$. Three classes of modes ($p-$, $g-$, and $c-$modes) have been identified (P97, and references therein). However, here we shall be concerned solely with radial ($m = 0$) $g-$modes, which are the most relevant observationally[4]. These $g-$modes are trapped where $\omega^2 < \kappa^2$, in the region where κ achieves its maximum value (at $r = 8\,GM/c^2$ for a nonrotating black hole) because, unlike in Newtonian gravity, $\kappa(r)$ rolls over at small r and vanishes at the inner disk edge, thereby creating a resonant cavity. The modes with the fewest radial nodes have relatively large radial extents $\Delta r \approx GM/c^2$.

The frequencies ($f = -\sigma/2\pi$) of the radial g–modes are

$$
\begin{aligned}
f \;&=\; 714 \;\left(\frac{M_\odot}{M}\right)\, F(a)\,[1-\epsilon_{nj}]\;\;\mathrm{Hz}\;, \\
\epsilon_{nj} \;&\sim\; \left(\frac{n+1/2}{j+\delta}\right)\left(\frac{h}{r}\right)\;. \qquad (1)
\end{aligned}
$$

(cf. P97). Here $F(a)$ is a known function of the dimensionless black hole angular momentum parameter $a = cJ/GM^2$, ranging from $F(0) = 1$ to $F(0.998) = 3.443$. The properties of the disk enter only through the small correction term involving the disk thickness $2h$, and the radial (n) and vertical (j) mode numbers, with $\delta \sim 1$ (as derived from the WKB solutions of P97). For thin disk models where $h/r \sim 0.1\; L/L_{Edd}$ (in the mode trapping region), ϵ_{nj} is typically on the order of a percent for the lowest modes ($n \sim j \sim 1$). Therefore, the mode frequency is relatively independent of disk luminosity. (The mode width, however, scales as $\Delta r \propto \sqrt{c_s} \propto \sqrt{h}$, where c_s is the sound speed in the disk.) The radial $g-$mode is thus a good candidate for explaining the recently discovered high-frequency features in the black hole candidate GRS 1915+105.

The observed frequency of this feature is 67 Hz, with a full width half maximum of $3-4$ Hz. (Morgan, Remillard, & Greiner 1996, 1997). This implies an effective $Q = f/\Delta f \sim 20$. Despite factors of ~ 2 luminosity variations in the source, the frequency remained constant to within $1-2$ Hz. The root mean square (rms) variability of the 67 Hz feature varied from $0.3\% - 1.6\%$ of the *total* observed X-ray luminosity, with the lower limit essentially being the Poisson noise limit. At its strongest, the rms variability $\sim 1\%$ below 6 keV, and the rms variability $\sim 6\%$ in the $12-25$ keV band, indicating that the spectrum of the feature is harder than the integrated source spectrum. (The spectrum of the source is fit by a power-law exponentially cut-off at ~ 5 keV.)

Below we discuss to what extent $g-$modes can explain these observations. In §2 we further review the properties of these modes, and make simple estimates of their luminosity modulation. In §3 we discuss application to GRS 1915+105. In §4 we discuss the role of turbulence and radiation as possible damping, growth, or excitation mechanisms. In §5 we present our conclusions.

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[4]These modes cover the largest area of the disk in a region where the disk temperature is maximum (away from the uncertain conditions at the disk edge).

2. LUMINOSITY MODULATION OF MODES

For both the psuedo-Newtonian (NW92, NW93) and fully relativistic (P97) diskoseismology calculations, the equations are formulated in terms of the potential $V \equiv \delta P/\rho$, where δP is the Eulerian variation of the pressure and ρ is the unperturbed density. For modes with an azimuthal and time dependence $\propto \exp[i(m\phi+\sigma t)]$, the potential $V(r,\eta,\phi,t)$ can be separated into a radial, $V_r(r)$, and a vertical, $V_\eta(\eta)$, component, where $\eta \equiv z/h(r)$ is the dimensionless vertical coordinate. The (small) fluid displacement vector, $\vec{\xi}$, can be related to the Eulerian potential $V(r,\eta,\phi,t)$ via the equations

$$\begin{aligned}\xi_r(r,\phi,z,t) &\approx (\omega^2-\kappa^2)^{-1}\ \exp[i(m\phi+\sigma t)]\ V_\eta \frac{\partial V_r}{\partial r}\\ \xi_z(r,\phi,z,t) &\approx (\omega^2-N_z^2)^{-1} \exp[i(m\phi+\sigma t)]\ V_r \frac{\partial V_\eta}{\partial z}\ ,\end{aligned} \quad (2)$$

where $N_z(\eta)$ is the vertical buoyancy frequency, which we shall set to zero (cf. P97 for a discussion of the role of non-vanishing bouyancy frequency). We shall usually use Δ to denote a *Lagrangian* variation of a given quantity, where we follow the definitions of NW92, NW93, and P97. (For $m=0$, $g-$modes have $\omega^2=\sigma^2 \lesssim \kappa^2$.)

The radial component of the fluid displacement approximately satisfies the WKB relation

$$\omega^2 c_s^2(r,0)\frac{d^2W}{dr^2} + (\omega^2-\kappa^2)(\omega^2-\Upsilon\Omega_\perp^2)\,W = 0\ , \quad (3)$$

where $\Upsilon(r)$ is a slowly varying separation function (akin to a separation constant in the WKB limit), $c_s(r,0)$ is the speed of sound at the disk midplane, $W \equiv (\kappa^2-w^2)^{-1}dV_r/dr$, and $\Omega_\perp = \Omega$ for a nonrotating black hole. We have taken the perturbations to be adiabatic, which means that the Lagrangian perturbation of the pressure and density are given by $\Delta P/P = \gamma\Delta\rho/\rho \approx -\gamma\vec{\nabla}\cdot\vec{\xi}$. Utilizing the radial WKB approximation employed in equation (3), the (approximate) components of the divergence become

$$\begin{aligned}\frac{\partial \xi_r}{\partial r} &\approx \left[(\Upsilon\Omega^2-\omega^2)/h^2\Omega^2\omega^2\right]\ \exp(i\sigma t)\ V_r(r)V_\eta(\eta)\\ \frac{\partial \xi_z}{\partial z} &\approx (h\omega)^{-2}\ \exp(i\sigma t)\ V_r(r)\frac{\partial^2 V_\eta(\eta)}{\partial\eta^2}\ .\end{aligned} \quad (4)$$

(We have also taken the unperturbed vertical barotropic structure to be in hydrostatic equilibrium.)

For most simple α-models, the energy generation rate per unit volume is approximately $\alpha P(r,z)\Omega(r)$. The modes not only perturb the pressure (we ignore possible perturbations to α, and one can show that perturbations to Ω are negligible), but they also perturb the locations of the disk boundaries. The variation of the luminosity is therefore

$$\begin{aligned}\delta L \sim 2\pi\Bigg[&\int_{r_I+\xi_r(r_I)}^{r_O+\xi_r(r_O)} r'dr' \int_{-z_0+\xi_z(-z_0)}^{z_0+\xi_z(z_0)} \alpha\Omega P'(r',z')dz'\\ &-\int_{r_I}^{r_O} rdr \int_{-z_0}^{z_0}\alpha\Omega P(r,z)\,dz\Bigg]\ ,\end{aligned} \quad (5)$$

where $P'(r',z') \equiv P(r,z)+\Delta P(r,z)$, $r' \equiv r+\xi_r(r)$, $z'=z+\xi_z(z)$, and r_I, r_O, z_0 are the disk boundaries (r_O can be taken to go to ∞ without loss of generality). Transforming variables, the change in luminosity can be written as

$$\begin{aligned}\delta L \sim\ & 2\pi\int_{r_I}^{r_O} dr\ \alpha\Omega(r) r\int_{-z_0}^{z_0} dz\ P(r,z)\\ \times\ & \Bigg\{(1-\gamma)\vec{\nabla}\cdot\vec{\xi} + \left[\frac{\partial\xi_r}{\partial r}\frac{\partial\xi_z}{\partial z} - \gamma\left(\vec{\nabla}\cdot\vec{\xi}\right)^2\right]\\ & - \gamma\frac{\partial\xi_r}{\partial r}\frac{\partial\xi_z}{\partial z}\vec{\nabla}\cdot\vec{\xi}\Bigg\}\ ,\end{aligned} \quad (6)$$

where we have employed the above expression for the Lagrangian pressure change, and employed the WKB approximation in *each* harmonic term above.

The luminosity fluctuation in general will have contributions from the fundamental frequency, as well as the first two harmonics. We have calculated the luminosity modulation for modes in a disk with radial sound speed profile equal to that of a radiation-pressure dominated Shakura-Sunyaev disk with $L/L_{Edd}=0.3$. The exact *radial* profile does not greatly effect the estimates, as we can replace the quantity $\int P(r,0)\Omega(r)dz$ with $F(r)$, the disk energy flux, which is known from energy conservation. As the mode width $\Delta r \propto \sqrt{h}$, the magnitude of h *does* affect the luminosity estimates. If we define the *maximum* magnitude (achieved at a radius, r_m) of the vertical displacement vector $\xi_z(r_m,z,t)$ to be $\equiv \mathcal{H}h(r_m)$, then for the case of the mode with one radial node in V_r and two vertical nodes in V_η ($\partial\xi_z/\partial\eta$ even about the midplane)[5] we have

$$\begin{aligned}\left(\frac{\delta L}{L}\right) \approx\ & 0.6\%\,\mathcal{H}\ \sin(\sigma t+\theta) - 9.8\%\,\mathcal{H}^2\ \sin^2(\sigma t+\theta)\\ & + 0.2\%\,\mathcal{H}^3\ \sin^3(\sigma t+\theta)\ .\end{aligned} \quad (7)$$

(Here we have used the psuedo-Newtonian approximation of NW92, NW93 for our calculations.) This is the modulation of the *bolometric* luminosity. As the mode exists in the inner, hotter regions of the accretion disk, the modulation in restricted high energy bandpasses will be greater. However, we have not properly accounted for radiative transfer effects. Dispersion in photon diffusion times over the extent of the mode will decrease the modulated luminosity for large optical depths.

3. APPLICATION TO GRS 1915+105

Although the above luminosity modulation is not large, $\mathcal{H}\sim 0.6-1.6$ produces, at the fundamental frequency, rms variability $\sim 0.3-1.6\%$ [$=(0.6\%\mathcal{H}+0.2\%\mathcal{H}^3)/\sqrt{2}$], as is seen for the feature in GRS 1915+105. The first harmonic can produce this observed rms variability for $\mathcal{H}\sim 0.3-0.7$. (The rms variability is the coefficient of the $\sin^2$ term divided by $\sqrt{8}$.) The modulation seen in GRS 1915+105 increases to 6% if the energy band is restricted to the $12-25$ keV range. This qualitatively agrees with what we expect for our modes. If we compare the bolometric luminosity variation of the mode to the bolometric luminosity from the region $r\approx 6-20\,GM/c^2$, the relative fractional variation increases to $\sim 6\%$.

For the *particular* radial and vertical disk structure assumed here, the rms variability in the first harmonic is more significant than the rms variability in the fundamental frequency. As

[5] The first “even” mode about the midplane is not trapped in the region of the epicyclic frequency maximum. The first “odd” mode leads to a vanishing vertical integral in the above estimate of luminosity modulation.

so little is known, both observationally and theoretically, about disk vertical profiles we cannot definitively state whether this feature is generic. Based upon energy arguments, however, we *can* say that generically disk modes can only produce rms variability $\sim \mathcal{O}(1\%)$ if $\mathcal{H} \sim 1$. We regard the question of which frequency, the fundamental or first harmonic, to associate with the observed 67 Hz feature as an open issue.

Equation (1) indicates that a 67 Hz (fundamental) modulation can be produced in a disk around a black hole with a mass of 10.6 $M_\odot$ for a nonrotating hole, and 36.3 $M_\odot$ for a maximally rotating hole. There are few "natural" frequencies to invoke in a black hole system. If one were to appeal to a process occuring at the marginally stable orbit ($r = 6\ GM/c^2$ for a nonrotating black hole), not only would the luminosity from this region be lower than our mode estimates (due to the no-torque boundary condition used in disk models), but also the required hole mass would increase by a factor of at least $16/(3\sqrt{3}) \sim 3$ (if the 67 Hz feature represents the fundamental mode frequency). Our nonrotating hole, fundamental frequency, estimate of the required mass would yield an observed X-ray luminosity (Morgan et al. 1997) $\gtrsim 30\%$ of the Eddington luminosity, consistent with our assumptions above.

4. MODE EXCITATION AND SELECTION EFFECTS

4.1. *Turbulent Damping and Excitation*

It is possible to use a parametrized stress tensor to estimate the effects of turbulent viscosity on the modes (NW92, NW93). The canonical energy of a radial mode is $E_c \sim \sigma^2 \rho (\xi_z^2 + \xi_r^2) \Delta V$, where ΔV is the volume occupied by the mode. Isotropic turbulence produces a rate of change $dE_c/dt \equiv -E_c/\tau$, with $\tau \sim |\ \alpha\sigma\ [h^2/\lambda_r^2 + h^2/\lambda_z^2]\ |^{-1}$ and λ_r, λ_z, respectively, being the radial and vertical mode wavelengths. The corresponding quality factor is given by

$$Q_{jn}{}^{-1} \ = \ (|\sigma|\tau)^{-1} \ \sim \ \left[j^2 + (h/r) n^2 \right] \ \alpha \ , \qquad (8)$$

as $\lambda_z \sim h/j$ and $\lambda_r \sim \sqrt{hr}/n$, where j and n are of order of the number of vertical and radial nodes in any particular eigenfunction. Thus, for $\alpha \ll 1$, we can have high mode Q.

The other contribution to the fractional width $\Delta f/f$ of the corresponding feature in a power spectrum comes from the number of modes that are significantly excited. If we assume that this will include those whose wavelengths are greater than the maximum eddy size $L \sim \alpha^{1/2} h$ divided by the corresponding mach number $\sim \alpha^{1/2}$, equation (1) gives

$$\Delta f/f \ \gtrsim \ \Delta n \, (h/r) \ \sim \ \sqrt{h/r} \ . \qquad (9)$$

The minimum effective frequency width will be composed of this span of modes, each broadened by $1/Q \sim \alpha$.

The above estimates are for *isotropic* viscosity. If the turbulence does not efficiently couple to the vertical gradients of the modes, then the mode Q value is increased by a factor $\sim (j\lambda_r/h)^2$ (NW93). Aside from damping modes, turbulence can also potentially excite modes. Velocity perturbations in the disk, $\delta\vec{v}$, are made up of a mode component, $\delta\vec{v}_M$, and a turbulent component, $\delta\vec{v}_T$. Viscous damping arises from terms of the form $\delta v_{M\,i} \delta v_{T\,j}$, while mode excitation arises from terms of the form $\delta v_{T\,i} \delta v_{T\,j}$ (NW93). It is possible to make simple estimates of the magnitude of the turbulent excitation, and balance this against the turbulent damping (NW93). The modes are excited to an amplitude of $|\xi^z| \sim \alpha (h/\lambda_r)^{3/2}\ h$, and $\sim \alpha \sqrt{\lambda_r/h}\ h$, for isotropic and anisotropic viscosity, respectively (NW93). If turbulence is playing the dominant role in damping and exciting the modes, then we have the following constraints. For isotropic turbulence, the Q value is only large for $\alpha \lll 1$; however, this implies a correspondingly small amplitude. For anisotropic viscosity, not only can we tolerate a larger α, but we also achieve a larger mode amplitude, although achieving the observationally required nonlinear mode amplitude is difficult in either case.

4.2. *Negative Radiative Damping*

As a first approximation, we took the modes to be adiabatic. In reality, we expect there to be small entropy changes due to various effects, the most notable one being radiative losses. If we have a radiation pressure dominated atmosphere, as is likely in high-luminosity disks, one properly should use

$$\Delta P \ = \ \gamma \frac{P}{\rho} \Delta\rho \ + \ \gamma \frac{P}{s} \Delta s \ , \qquad (10)$$

where s is the specific entropy. We can estimate[6] the effect of this term for a radiation pressure dominated atmosphere, where one has

$$4 \frac{P}{s} \frac{Ds}{Dt} \ \approx \ -\frac{c}{\kappa_{es}} \vec{\nabla} \cdot \frac{\vec{\nabla} P}{\rho} \ \approx \ \frac{c}{\kappa_{es}} \Omega^2 \qquad (11)$$

(c is the speed of light, κ_{es} is the electron scattering opacity, and we have invoked vertical hydrostatic equilibrium.)

For perturbations of scalars, the Lagrangian perturbation operator Δ commutes with total time derivatives D/Dt (cf. Lynden-Bell & Ostriker 1967). Using this fact, and the fact that for our modes $D\Delta s/Dt = i\omega\Delta s$, we can show that

$$\begin{aligned} \frac{P\Delta s}{s} \ &= \ \left(1 - i \frac{4\kappa_{es} P \omega}{c\Omega^2} \right)^{-1} \left(\Delta P - P\xi_r \frac{\partial \ln \Omega^2}{\partial r} \right) \\ &= \ \left(1 - i \ \frac{4\omega\tau_{es} h}{c} \right)^{-1} \left(\Delta P - P\xi_r \frac{\partial \ln \Omega^2}{\partial r} \right) , \end{aligned} \qquad (12)$$

where τ_{es} is the electron scattering optical depth of the atmosphere. The term proportional to $\partial \ln \Omega^2/\partial r$ in the above is negligible in the WKB approximation. The Lagrangian variation of the entropy is therefore directly proportional to the Lagrangian variation of the pressure.

We do not know the true radial and vertical structure of a disk model that correctly describes the spectral observations of GRS 1915+105. However, for a "standard" Shakura-Sunyaev $\alpha-$disk model, one can show, with $\omega \sim \kappa$, that $\omega\tau_{es} h/c \sim \alpha^{-1}$, and hence is likely to be $\gg 1$. Taking this to be the case, we can combine equation (10) and equation (12) to yield $\Delta P/P \equiv \gamma' \Delta\rho/\rho$, where

$$\gamma' \ \approx \ \gamma \left(1 \ + \ i \frac{\gamma c}{4\omega\tau_{es} h} \right) \ \equiv \ \gamma \, (1 \ + \ i\alpha') \ . \qquad (13)$$

We have subsumed our ignorance of the disk's vertical structure into the parameter α' which we expect to be of $\mathcal{O}(\alpha)$.

[6] Throughout this section our estimates shall be based upon vertically averaged quantities rather than fully $z-$ dependent quantities.

To estimate the effects that the non-adiabatic terms have upon our modes, we can substitute the above into equation (3) with $\partial^2 W/\partial r^2 \approx -k_r^2 W$ and then expand it about the adiabatic solution. Specifically, we then have a dispersion relation that can be written in terms of a function $G(\omega) = 0$, to which we are adding a function $H(\omega)$ due to radiation damping. If ω_0 is the unperturbed mode frequency and $\delta\omega$ is its perturbation, we then have to first order

$$\left.\frac{\partial G}{\partial \omega}\right|_{\omega_0} \delta\omega \; + \; H(\omega_0) \; = \; 0 \; . \tag{14}$$

The mode Q value then becomes

$$Q \; \equiv \; i\frac{\omega}{\delta\omega} \; = \; -i\frac{\partial G/\partial \ln \omega|_{\omega_0}}{H(\omega_0)} \; \sim \; -\frac{2\Upsilon}{\alpha'}\left(\frac{\Omega}{c_s k_r}\right)^2 , \tag{15}$$

where on the right side of the above we have used equation (3), and have taken the term $\Upsilon\Omega_\perp$ to be dominant (i.e. $\Upsilon \gtrsim 1$). The minus sign indicates mode *growth*; that is, the modes are *unstable* to radiative losses. As typically $k_r^{-1} > h$, $\Upsilon > 1$, and $\alpha < 1$, it is fairly easy to obtain $|Q| \gg 1$.

4.3. *Selection Effects*

As discussed in §2, the modes must approach nonlinearity in order to be observationally relevant. If isotropic turbulent excitation determines the mode amplitude, then there is a natural maximum vertical perturbation amplitude with $\mathcal{H} \ll 1$. If the turbulence is anisotropic, on the other hand, then it is possible to achieve $\mathcal{H} \sim 1$, so long as $\alpha \sim \sqrt{h/\lambda_r} \lesssim 1$. For this case we expect Q values $\sim \alpha^{-3}$. Radiative excitation also naturally leads to $\mathcal{H} \sim 1$. At some point, currently unknown, damping effects and nonlinearities must limit this radiative mode growth. As a zeroth-order approximation we take the linear growth rates calculated above to be relevant to the near nonlinear regime, and thus be equal to the saturated damping rate. The above radiative Q value then gives a rough estimate of the width of observationally relevant modes.

The low rms variability of the modes means that only modes with high Q values will be detectable. Observational limits for $Q \sim 20$ were rms $\sim 0.3\%$, and detectable rms $\propto Q^{-1/2}$ for narrow modes. Considering the dominant contributions as n and j increase, turbulence leads to $Q \propto j^{-2}$ (isotropic) and $Q \propto n^{-2}$ (anisotropic), whereas radiative damping leads to $Q \propto j^2/n^2$ The limiting detectable rms variabilty is therefore linear in n and j. Turbulent excitation only leads to high Q values for modes with low j (isotropic) or low n (anisotropic). Radiative damping, on the other hand, only leads to high Q values for modes modes with low n and high j. We note, however, that the mode calculations require a smooth, well-behaved background to perturb. If small scale turbulence truly is responsible for disk viscosity, our unperturbed hydrodynamic equations are only relevant for the largest scales, again favoring both low n and low j. For all cases considered above, a high Q value is most easily achieved for $\alpha \ll 1$. The 67 Hz mode seen in GRS 1915+105, if a diskoseismic mode, therefore is likely one with low n and j, and $\alpha \ll 1$.

5. CONCLUSIONS

The main motivations for attributing the 67 Hz feature in GRS 1915+105 to a diskoseismic mode are that these modes: 1) are related to a "natural" frequency in the disk (i.e. the maximum epicyclic frequency); 2) their spectra are expected to be characteristic of the inner, hottest regions of the disk; 3) their frequencies are relatively insensitive to changes in luminosity; and 4) they have low rms variability. This latter feature, although in agreement with the observations, is the strongest constraint. These modes *cannot* be applied to systems that show $\gtrsim 10\%$ rms variability over a wide range of energy bands. We have identified two mechanisms, turbulent excitation and negative radiation damping, that naturally lead to appreciable mode amplitudes with high Q value for $\alpha \ll 1$.

Again, we stress that the only other "natural" disk frequency, the Keplerian rotation frequency at the last marginally stable circular orbit, would require a black hole mass a factor $\gtrsim 3$ larger. Furthermore, less flux is emitted from that region than from the $g-$mode region.

We saw that there were two requirements for the $g-$modes to be observationally detectable. First, the $g-$modes had to be wide, or equivalently the disk had to be luminous, assuming that $\sqrt{c_s} \propto \sqrt{L}$. Second, the $g-$mode amplitudes had to approach the nonlinear regime. Ideally, one should perform numerical simulations of the nonlinear development of the $g-$modes to determine mode widths, Q, and luminosity modulation more accurately. It is interesting to note that the MHD simulations of Stone et al. (1996) do show copious $p-$mode production associated with the turbulence (Gammie 1996). (MHD simulations have yet to be performed for rotation and epicyclic frequency profiles relevant to the $g-$mode trapping region.)

These $g-$mode oscillations should also exist in accretion disks around compact (soft equation of state), weakly magnetized ($B < 10^8$ gauss) neutron stars. Under these conditions, the inner radius of the disk will be less than those radii where the $g-$mode exists. Again, large amplitude (rms $\gg$ 1%) features *cannot* be explained with these modes.

Even if the 67 Hz feature seen in GRS 1915+105 does not turn out to be a diskoseismic mode, it points out two important lessons. First, BHC systems can produce relatively stable, high-frequency features. Second, the *Rossi X-ray Timing Explorer* is capable of detecting and characterizing these features despite their weak variability. The search for diskoseismic modes in this system and other BHC has therefore become a viable and worthwhile pursuit.

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REFERENCES

Gammie, C. 1996, private communication
Lynden-Bell, D., & Ostriker, J.P. 1967, MNRAS, 136, 293
Morgan, E., Remillard, R., & Greiner, J. 1996, IAUC 6392
———. 1997, ApJ, submitted
Nowak, M.A., & Wagoner, R.V. 1992, ApJ, 393, 697 (NW92)
Nowak, M.A., & Wagoner, R.V. 1993, ApJ, 418, 187 (NW93)
Perez, C.A., Silbergleit, A.S., Wagoner, R.V., & Lehr, D.E. 1997, ApJ, 476, in press (P97)
Stone, J.M., Hawley, J.F., Gammie, C.F., & Balbus, S.A. 1996, ApJ, 463, 656